Robust control using linear quadratic method in hybrid-electromagnetic suspension system

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ABSTRACT: Hybrid-electromagnetic suspension (EMS) system is type that vehicle levitates from guide-way. An advantage of this system is no energy consumption at nominal operating point which is maintains levitation using attraction force by permanent magnetic only. The EMS system is inherently unstable. It means active control needs to be incorporated for stable levitation. And it needs control method which is satisfied stability and robustness when disturbance is existed. This paper presents robust control using linear quadratic (LQ) method in hybrid-EMS system. We simulate dynamic characteristics using pole replacement method and LQ method. The proposed method is better dynamic response by computer simulation.

1 INTRODUCTION

Railway among land transport has contributed a lot in human life and industrial development for the latest in a century. However share of railway was much relatively lower in passenger and freight transport because of rapid spread of car. But railway is effective and efficient system in terms of environmental.

Magnetically Levitated Vehicle (MAGLEV) is getting spotlight newly in railway transport. MAGLEV is driving system without contact with rails by electromagnetic force. Magnetic levitation system is divided by electrodynamic suspension (EDS) and electromagnetic suspension (EMS).

A lot of researchers have been interested in hybrid-EMS in term of energy saving. EMS system has inherently unstable levitation characteristics with nonlinearity and parameter variation. Levitation controllers must maintain stability with gap variation and disturbance and have excellent dynamic response.

State feedback controller which is traditional method using PID or pole placement is difficult to precise and robust control with gap variation and disturbance. So a lot of studies proposed for robust control.

This paper presents robust control using linear quadratic (LQ) method in hybrid-EMS system. The proposed method is better dynamic response by computer simulation.

2 MATHEMATICAL MODEL OF HYBRID-EMS SYSTEM

2.1 *Magnetic Equivalent Circuit of Hybrid-EMS System*

Figure 1 shows magnetic equivalent circuit of hybrid-EMS system. It assumes no magnetic saturation and leakage flux.

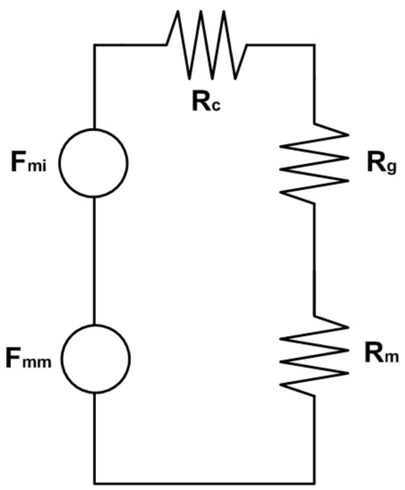

Figure 1. Magnetic equivalent circuit of hybrid electromagnetic suspension system

Total reluctance is given by Equation 1.

$$R_m = R_g + R_c + R_p \tag{1}$$

where, R_m is total reluctance, R_g, R_c, R_p are reluctance of air gap, core and permanent magnet.
Each reluctances are following.

$$R_g = \frac{l_g}{\mu_0 A}, \quad R_c = \frac{l_c}{\mu_c A}, \quad R_p = \frac{l_p}{\mu_p A} \tag{2}$$

Magnetomotive force is given by Equation 3 and 4.

$$F_{mi} = Ni(t) \tag{3}$$

$$F_{mm} = \Phi_m(t) R_p = (B_r A)\frac{l_p}{\mu_p A} = \frac{B_r l_p}{\mu_p} \tag{4}$$

where, F_{mi}, F_{mm} is magnetomotive force of electromagnet and permanent magnet, N is turn number of coil, $i(t)$ is current of coil, $\Phi_m(t)$ is flux of permanent magnet.

Using Equation 1~4, flux and flux density is given.

$$\Phi(t) = \frac{F_{mm} + F_{mi}}{R_c + R_g + R_p} = \frac{\dfrac{B_r \cdot l_p}{\mu_p} + Ni(t)}{\dfrac{l_c}{\mu_c A} + \dfrac{2z(t)}{\mu_0 A} + \dfrac{l_p}{\mu_p A}} \tag{5}$$

$$B(t) = \frac{\Phi(t)}{A} = \frac{\dfrac{B_r \cdot l_p}{\mu_p} + Ni(t)}{\dfrac{l_c}{\mu_c} + \dfrac{2z(t)}{\mu_0} + \dfrac{l_p}{\mu_p}} \tag{6}$$

If l_c / μ_c is neglected, flux density is rewritten.

$$B(t) = \frac{\Phi(t)}{A} = \frac{\dfrac{B_r \cdot l_p}{\mu_p} + Ni(t)}{\dfrac{2z(t)}{\mu_0} + \dfrac{l_p}{\mu_p}} = \frac{\left(\dfrac{B_r \cdot l_p}{\mu_p} + Ni(t)\right) \times \dfrac{\mu_p}{l_p}}{\left(\dfrac{2z(t)}{\mu_0} + \dfrac{l_p}{\mu_p}\right) \times \dfrac{\mu_p}{l_p}} \tag{7}$$

Inductance at air-gap is given by equation (8).

$$L(z) = \frac{N^2}{R_m} = \frac{N^2}{\dfrac{l_c}{\mu_c A} + \dfrac{2z(t)}{\mu_0 A} + \dfrac{l_p}{\mu_p A}} = \frac{N^2 A}{\dfrac{l_c}{\mu_c} + \dfrac{2z(t)}{\mu_0} + \dfrac{l_p}{\mu_p}} \tag{8}$$

Attraction force of air-gap is following equation.

$$F_a(t) = \frac{B^2(t)}{2\mu_0} \times 2A = \frac{A}{\mu_0} \times \left(\frac{B_r + \beta i(t)}{1 + \alpha z(t)}\right)^2 \tag{9}$$

2.2 Voltage and Dynamic Equation of Hybrid-EMS System

Voltage equation of hybrid-EMS system is given by equation (10).

$$\begin{aligned} e_a(t) &= Ri(t) + \frac{d\Phi(t)}{dt} \\ &= Ri(t) + \frac{d}{dt}\left[L(z,i)i(t)\right] \\ &= Ri(t) + L(t)\frac{di(t)}{dt} + i(t)\frac{dL(z)}{dz}\frac{dz(t)}{dt} \\ &= Ri(t) + \frac{\mu_0 N^2 A}{2z(t) + \dfrac{l_p}{1.05}}\frac{di(t)}{dt} - \frac{\mu_0 N^2 A i(t)}{\left[2z(t) + \dfrac{l_p}{1.05}\right]^2}\frac{dz(t)}{dt} \end{aligned} \tag{10}$$

We could attain the equation (11) by reorganizing the equation (10) in terms of current perturbation.

$$\begin{aligned} \frac{di(t)}{dt} &= -\frac{\left(2z(t) + \dfrac{l_p}{1.05}\right)}{\mu_0 N^2 A} Ri(t) + \frac{i(t)}{\left(2z(t) + \dfrac{l_p}{1.05}\right)}\frac{dz(t)}{dt} \\ &\quad + \frac{\left(2z(t) + \dfrac{l_p}{1.05}\right)}{\mu_0 N^2 A} e_a(t) \end{aligned} \tag{11}$$

Dynamic equation of hybrid-EMS system is given by equation (12).

$$\begin{aligned} M\frac{d^2 z(t)}{dt^2} &= Mg - F_a(t) + F_d(t) \\ &= Mg - \frac{A}{\mu_0}\left(\frac{B_r + \beta i(t)}{1 + \alpha z(t)}\right)^2 + F_d \end{aligned} \tag{12}$$

$$\frac{d^2 z(t)}{dt^2} = g - \frac{1}{M}\frac{A}{\mu_0}\left(\frac{B_r + \beta i(t)}{1 + \alpha z(t)}\right)^2 + \frac{1}{M}F_d(t) \tag{13}$$

Assuming no disturbance, dynamic equation of hybrid-EMS system at nominal operating point is given by equation (14).

$$Mg = F_0 = \frac{A}{\mu_0}\left(\frac{B_r + \beta i_0}{1+\alpha z_0}\right)^2 \tag{14}$$

By using linear approximation for excursions around the nominal operating point (i_0, z_0), reasonably accurate linear model be obtained. The small perturbation linear equations of system are expressed equation (15), (16) and (17).

$$\begin{aligned} e_0 + \Delta e_a(t) &= R(i_0 + \Delta i(t)) + \frac{\mu_0 N^2 S}{\left(2z_0 + \frac{l_p}{1.05}\right)} \frac{d\Delta i(t)}{dt} \\ &\quad - \frac{\mu_0 N^2 S i_0}{\left(2z_0 + \frac{l_p}{1.05}\right)^2}\left(\frac{d\Delta z(t)}{dt}\right) \end{aligned} \tag{15}$$

$$\begin{aligned} \frac{d\Delta i(t)}{dt} &= -\frac{\left(2z_0 + \frac{l_p}{1.05}\right)}{\mu_0 N^2 S} R(i_0 + \Delta i(t)) \\ &\quad + \frac{i_0}{\left(2z_0 + \frac{l_p}{1.05}\right)} \frac{d\Delta z(t)}{dt} \\ &\quad + \frac{\left(2z_o + \frac{l_p}{1.05}\right)}{\mu_0 N^2 S}(e_0 + \Delta e_a(t)) \end{aligned} \tag{16}$$

$$M\frac{d^2}{dz^2}(z_0 + \Delta z(t)) = Mg - (F_0 + \Delta F_a(t)) + F_d(t) \tag{17}$$

Perturbation of attraction force is expressed by current and air-gap partial differential equation.

$$\begin{aligned} M\frac{d^2\Delta z(t)}{dt^2} &= Mg - \frac{A}{\mu_0}\left(\frac{B_r + \beta i_0}{1+\alpha z_0}\right) - \frac{\partial F_a(t)}{\partial i(t)}\Delta i(t) \\ &\quad - \frac{\partial F_a(t)}{\partial z(t)}\Delta z(t) + F_d(t) \end{aligned} \tag{18}$$

Because gravity is equal to attraction force at nominal operating force, equation (18) is expressed equation (19).

$$M\frac{d^2}{dz^2}\Delta z(t) = -\frac{\partial F_a(t)}{\partial i(t)}\Delta i(t) - \frac{\partial F_a(t)}{\partial z(t)}\Delta z(t) + F_d(t) \tag{19}$$

$$\frac{d^2\Delta z(t)}{dt^2} = -\frac{c_{ih}}{M}\Delta i(t) + \frac{c_{zh}}{M}\Delta z(t) + \frac{1}{M}F_d(t) \tag{20}$$

Where, $c_{ih} = \dfrac{2A\beta B_r}{\mu_0(1+\alpha z_0)^2}$, $\quad c_{zh} = \dfrac{2A\alpha B_r^2}{\mu_0(1+\alpha z_0)^3}$

2.3 State-space Equation of Hybrid-EMS System

Using Voltage and dynamic equation, state-space equation is given by equation (21).

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{c_{zh}}{M}x_1 - \frac{c_{ih}}{M}x_3 + \frac{1}{M}F_d(t) \\ \dot{x}_3 &= -\frac{R}{L_0}x_3 + \frac{1}{L_0}\Delta e_a(t) \end{aligned} \tag{21}$$

where, each parameter matrix of state-space equation are followings.

$$\mathbf{x} = \left[\Delta z(t)\ \ \Delta v(t)\ \ \Delta i(t)\right]^T = \left[x_1\ \ x_2\ \ x_3\right]^T \tag{22}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{c_{zh}}{M} & 0 & -\frac{c_{ih}}{M} \\ 0 & 0 & -\frac{R}{L_0} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M} \\ \frac{1}{L_0} & 0 \end{bmatrix}, \tag{23}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\mathbf{u} = \left[\Delta e_a(t)\ F_d(t)\right]^T \tag{24}$$

Figure 2 shows block diagram of state-space equation of hybrid EMS system.

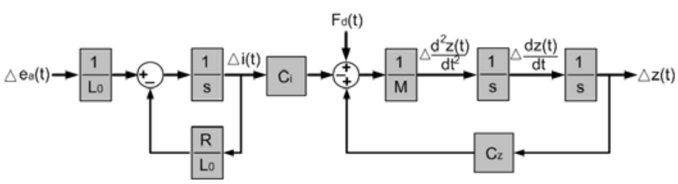

Figure 2. block diagram of state-space equation of hybrid EMS system.

2.4 Characteristic of Hybrid-EMS System

It assumes that initial conditions of state-space and output equations are zero. Equation (25) is obtained by Laplace transformation.

$$\frac{\Delta z(s)}{\Delta e_a(s)} = \frac{-c_{ih} \times \dfrac{1}{L_0 M}}{s^3 + \dfrac{R}{L_o} s^2 - \dfrac{c_{zh}}{M} s - \dfrac{c_{zh} R}{L_0 M}} \tag{25}$$

For getting poles and zeros, It uses root-locus. As a result, two poles are left half plane, and one pole is right half plane. Because of one pole of right half plane, system is unstable.

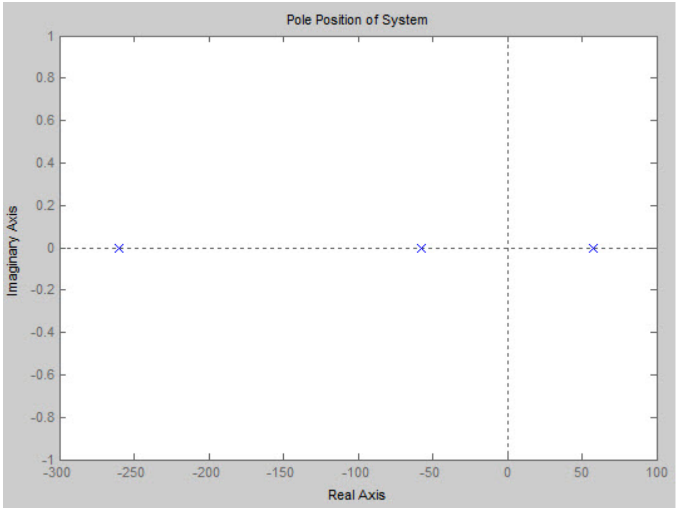

Figure 3 Position of open-loop poles in hybrid-EMS system.

3 ROBUST CONTROL OF LEVITATION SYSTEM

3.1 Conventional Control Method(Pole placement)

Consider a plant represented in state-space by equation (26)

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y &= \mathbf{C}\mathbf{x} + Du \end{aligned} \tag{26}$$

Control signal is given by equation (27)

$$u = -\mathbf{K}\mathbf{x} \tag{27}$$

Feedback through the gains is represented in Figure 4 by feedback vector $\mathbf{K}$. Combining equation 26 and 27, equation 28 is obtained.

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} \tag{28}$$

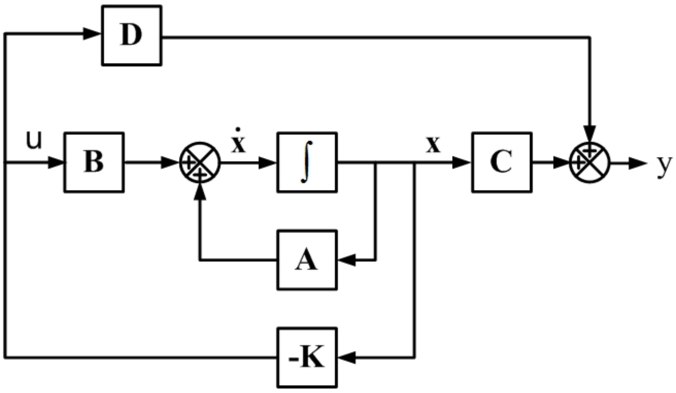

Figure 4. State-space representation of a plant with state-variable feeback.

State-space equation of Hybrid-EMS system with current error integrator is given by equation (29)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{c_{zh}}{M} & 0 & -\dfrac{c_{ih}}{M} \\ 0 & 0 & -\dfrac{R}{L_0} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{1}{M} \\ \dfrac{1}{L_0} & 0 \end{bmatrix}, \tag{29}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\mathbf{u} = \left[\Delta e_a(t)\ F_d(t)\right]^T = \left[-\mathbf{K}\mathbf{x} + K_I x_e\ F_d(t)\right]^T$$

where, $\dot{x}_e$ is error of current, K_I is gain of current error integrator.

Transfer function is given by equation (30) using equation (29)

$$G(s) = \mathbf{C}(sI - \mathbf{A})^{-1}\mathbf{B} = \frac{K_I\left(s^2 M - c_z\right)/ML_0}{\vartriangle s} \tag{30}$$

where, $\vartriangle s = s^4 + a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0$ and each parameter is

$$a_3 = \frac{R + K_c}{L_0},\ a_2 = \frac{MK_I - c_i K_v - c_z L_0}{ML_0},$$
$$a_1 = -\frac{c_z(R + K_c) - c_i K_z}{ML_0},\ a_0 = -\frac{c_z K_I}{ML_0}$$

We set settling time is 0.4[s] and percent overshoot is 1[%]. Using pole replacement, feedback gains are obtained.

$$K_c = 9.335,\ K_v = -1.9605 \times 10^3,$$
$$K_z = -7.7720 \times 10^4,\ K_I = -119.1553$$

Figure 4 shows position of close-loop poles in hybrid-EMS system using pole replacement.

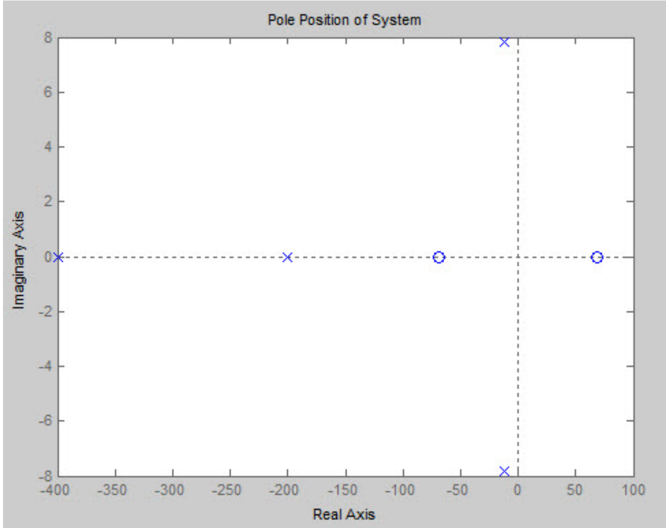

Figure 4 Position of open-loop poles in hybrid-EMS system using pole replacement

3.2 *Proposed Control Method (Linear quadratic)*

If a plant represented in state-space by equation (26) and control is given by equation (27), cost function is given by equation (31).

$$J = \int_0^\infty \left[\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t) \right] dt \qquad (31)$$

An object of LQ method is that find matrix $\mathbf{K}$ which makes minimum of cost function. As a result, control signal is satisfied Riccati equation.

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = 0 \qquad (32)$$

Using LQ method, feedback gains are obtained.

$$K_c = 312.9822,\ K_v = -3.3290 \times 10^4,$$
$$K_z = -1.9264 \times 10^6,\ K_I = 3.8730 \times 10^3$$

Figure 5 shows position of close-loop poles in hybrid-EMS system using LQ method.

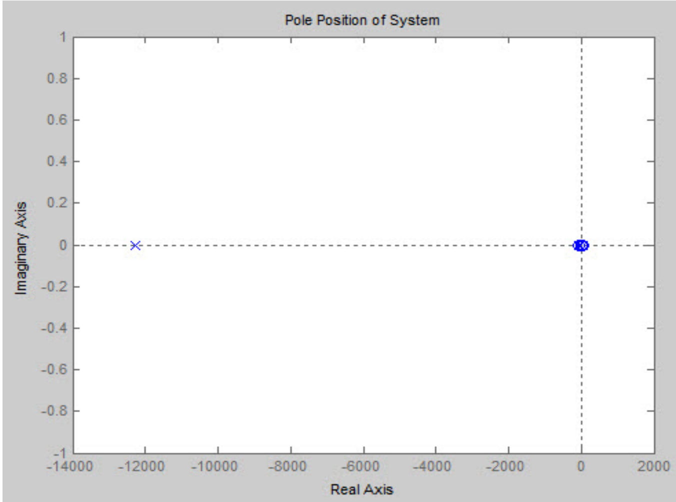

Figure 5 Position of open-loop poles in hybrid-EMS system using LQ method

4 SIMULATION

Table 1 shows specification of hybrid-EMS system. Initial air-gap is 7.5[mm], nominal operating air-gap is 5[mm].

Table 1. Specification of hybrid-EMS system.

Item		Unit	Value
Core	Total Height	mm	47
	Total Width	mm	60
	Teeth Height	mm	33
	Teeth width	mm	10
Hybrid electromagnet	Weight	kg	3.5
	Turns of coil		660
	Permanent magnet		Nd-Fe-B (N42)
	Height of PM	mm	14
	Area of PM	mm^2	700
	Flux density	T	1.27

4.1 *Without Disturbance*

4.1.1 *Pole replacement*

Figure 6 ~ 10 shows current, air-gap, acceleration and velocity characteristic.

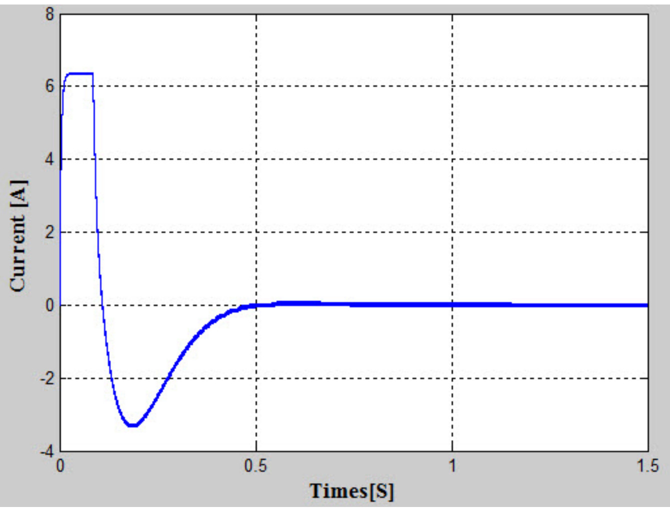

Figure 6. Current characteristic with pole replacement.

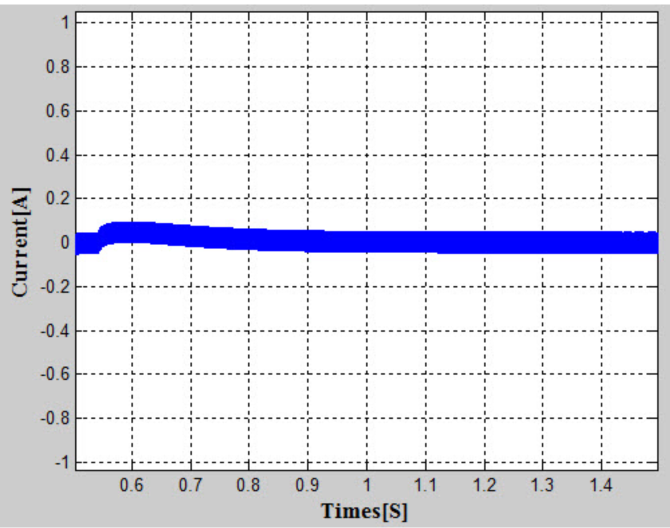

Figure 7. .Current characteristic with pole replacement at steady state.

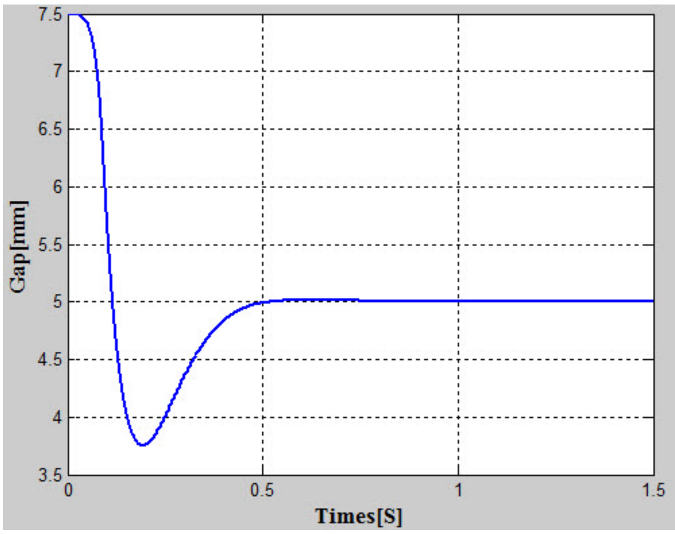

Figure 8. Air-gap characteristic with pole replacement.

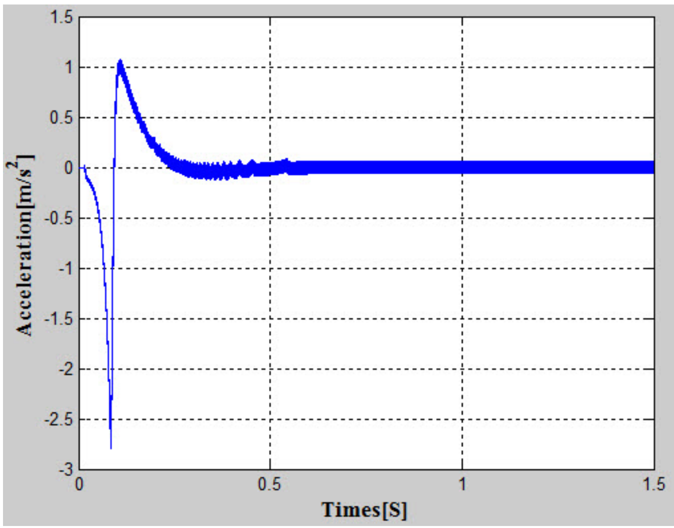

Figure 9. Acceleration characteristic with pole replacement.

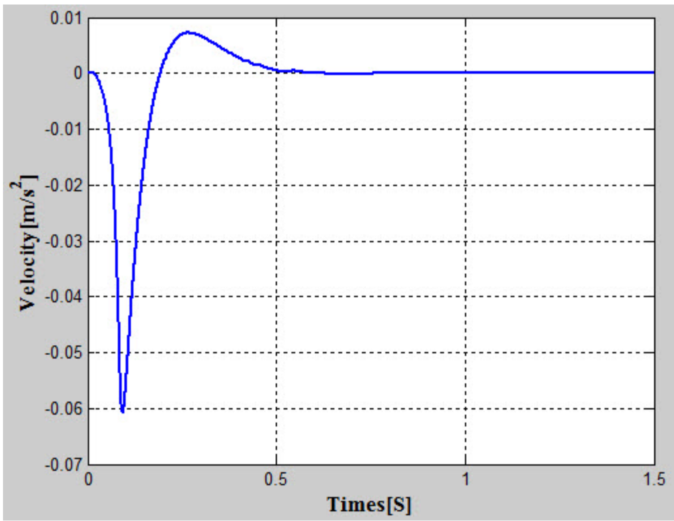

Figure 10. Velocity characteristic with pole replacement.

4.1.2 Linear Quadratic Method

Figure 11 ~ 15 shows current, air-gap, acceleration and velocity characteristic.

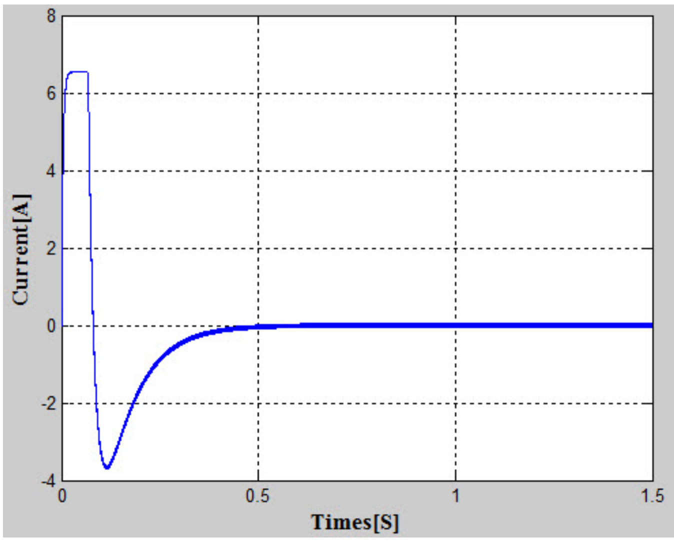

Figure 11. Current characteristic with LQ method.

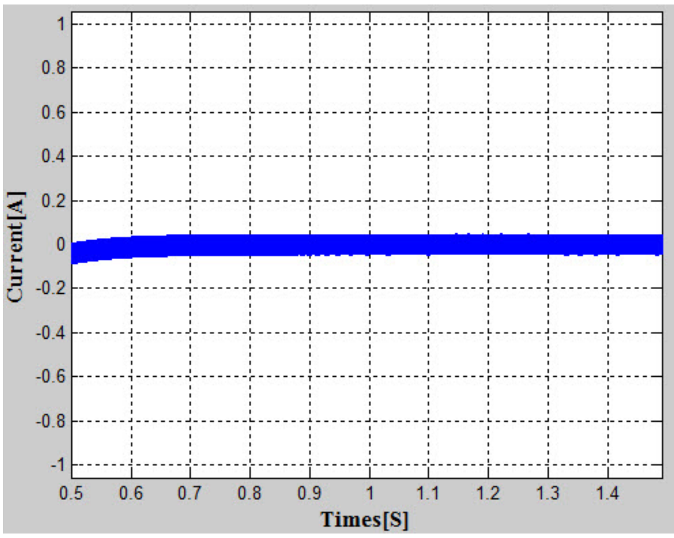

Figure 12. .Current characteristic with pole LQ method at steady state.

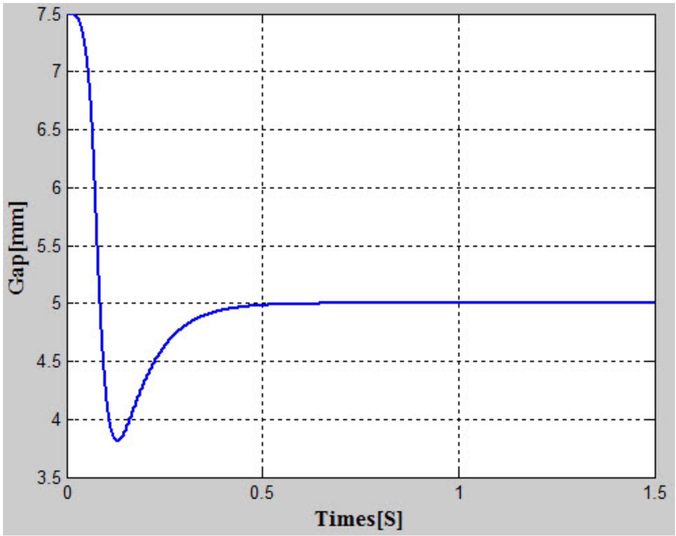

Figure 13. Air-gap characteristic with LQ method.

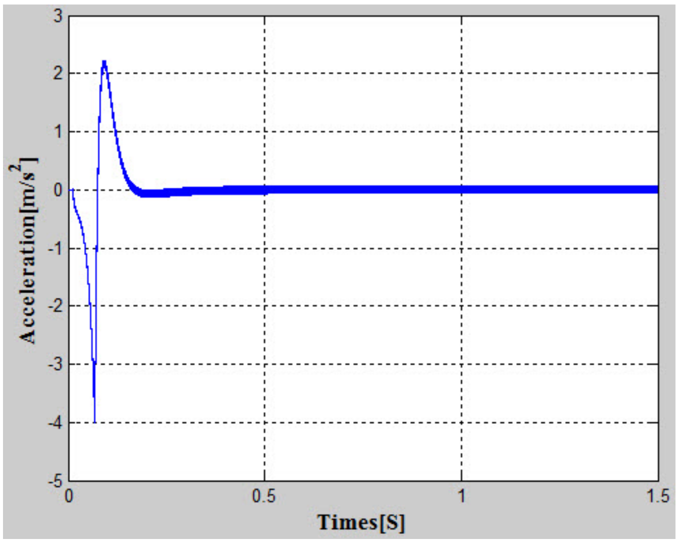

Figure 14. Acceleration characteristic with LQ method.

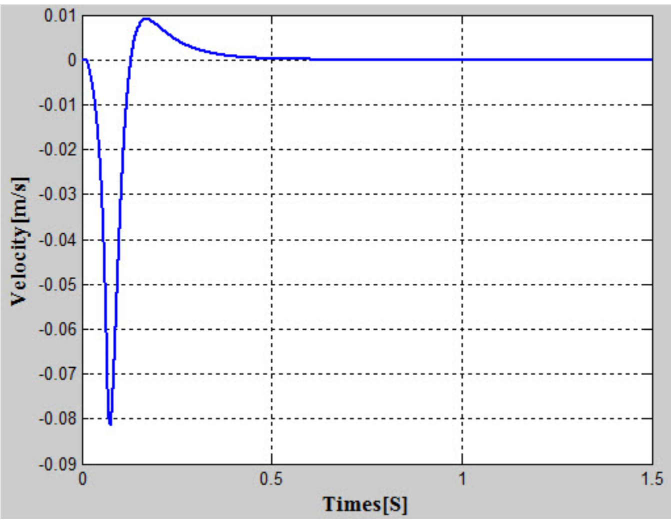

Figure 15. Velocity characteristic with LQ method.

4.2 *With Disturbance*

Figure 16 shows disturbance profile.

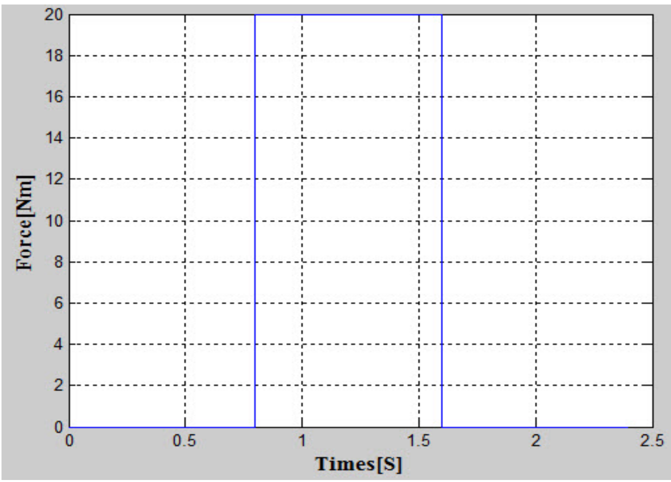

Figure 16. Disturbance profile.

4.2.1 Pole replacement

Figure 17 ~ 20 shows current, air-gap, acceleration and velocity characteristic.

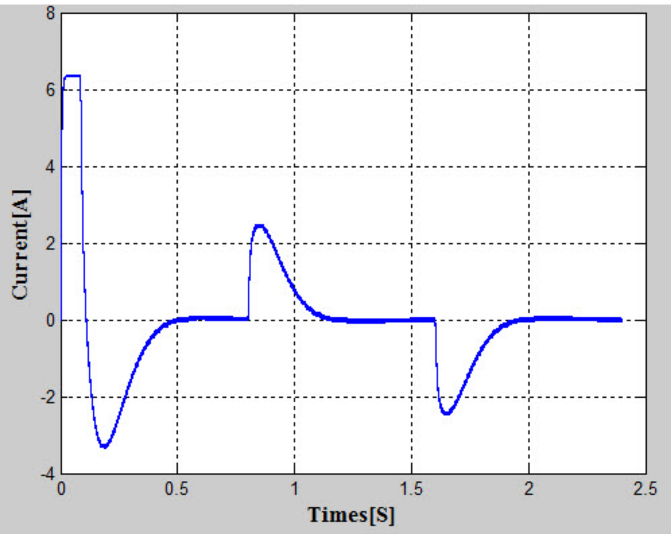

Figure 17. Current characteristic with pole replacement.

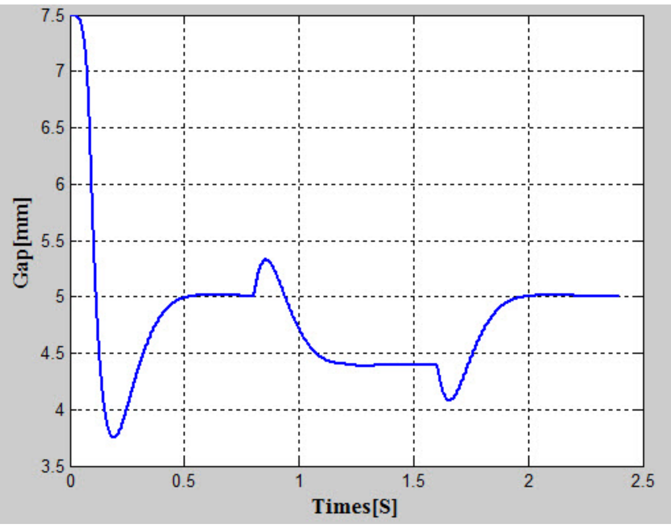

Figure 18. Air-gap characteristic with pole replacement.

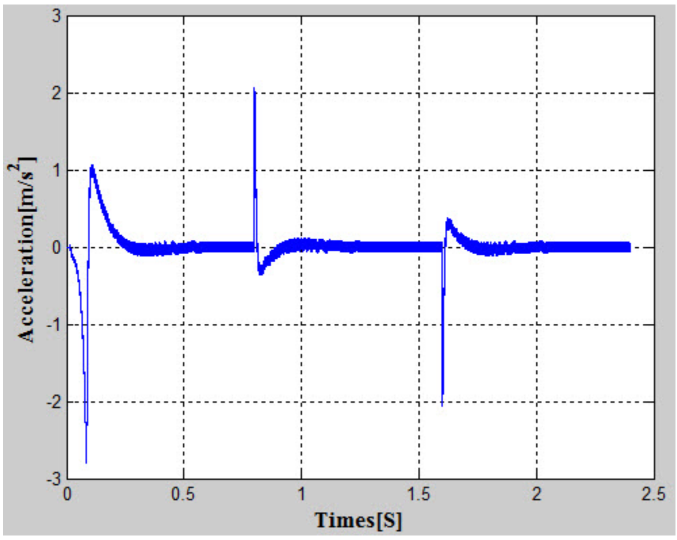

Figure 19. Acceleration characteristic with pole replacement.

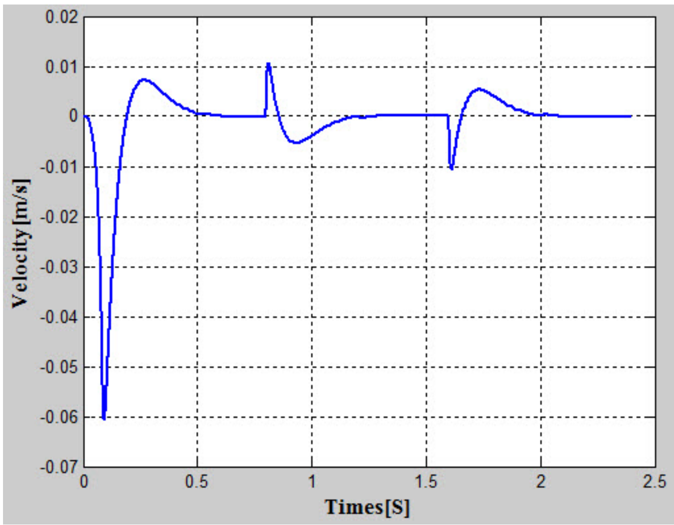

Figure 20. Velocity characteristic with pole replacement.

4.2.2 Linear Quadratic Method

Figure 21 ~ 24 shows current, air-gap, acceleration and velocity characteristic.

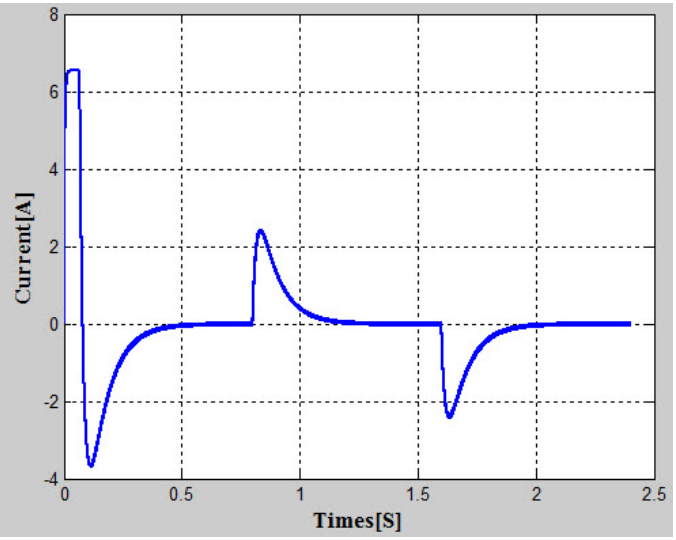

Figure 21. Current characteristic with LQ method.

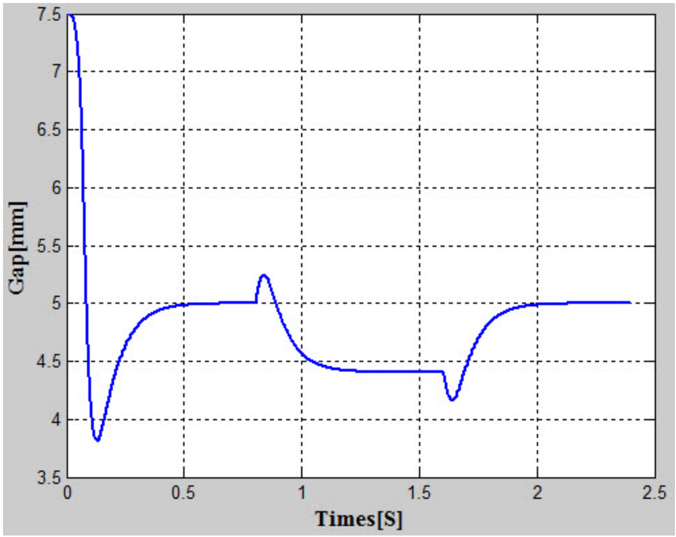

Figure 22. Air-gap characteristic with LQ method.

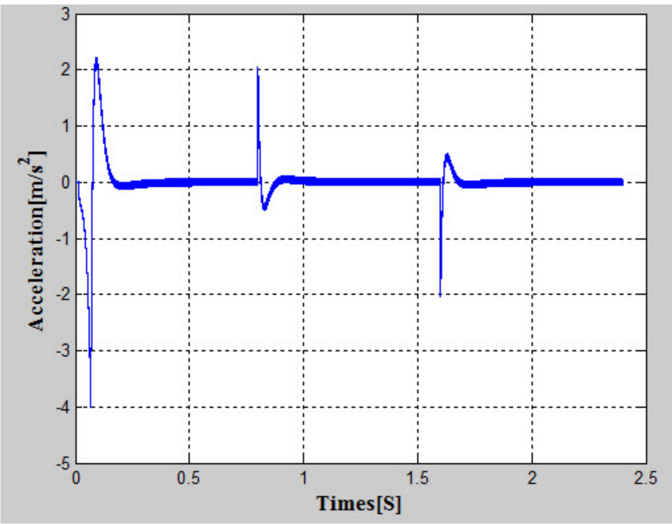

Figure 23. Acceleration characteristic with LQ method.

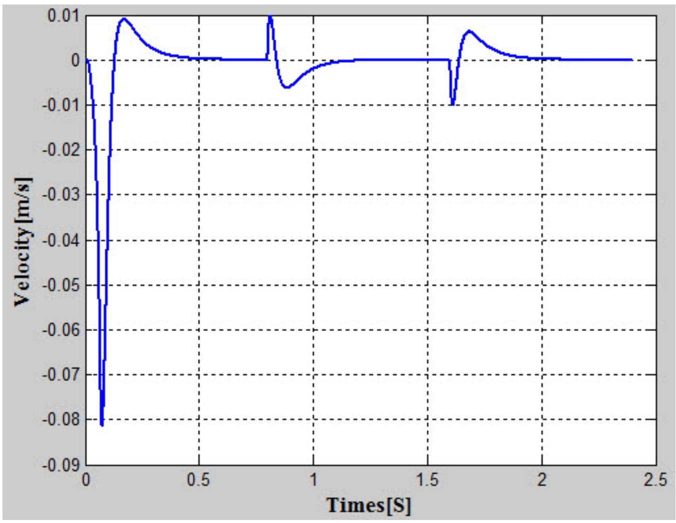

Figure 24. Velocity characteristic with LQ method.

5 CONCLUSIONS

We propose controller using LQ method to control levitation system. Optimal control theory is focused dynamic system is operating at a minimum cost.

In simulation, dynamic characteristic of LQ controller is better than traditional controller without disturbance. That is, settling time is decreased and current ripple of steady state is reduced.

Also control of LQ controller is more robust than traditional control. When applied to disturbance, LQ controller's current and gap variation is smaller than traditional controller.

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